Odisha Board Class 12 Maths Sample Paper

Class: XII

Subject: Mathematics

Total: 100 Marks

General Instructions:

1. All questions are compulsory in Group A, which are very short answer type questions. All questions in the group are to be answered in one word, one sentences are as per exact requirement of the question.

2. Group – B contain 5 questions and each question have 5 bits, out of which only 3 bits are to be answered.

3. Group- C contains 5 questions and each question have 2 or 3 bits, out of which only 1 bit is to be answered. Each bit carries 6 marks.

GROUP-A

Total Marks: 10

1. Find the domain $f(x) = \frac{3}{\sqrt{[x]^2-3[x]-10}}$

2. Maximize the value of $4x + 5y = z$, Constraints are $2x + 3y \leq 6$ and $3x + y \leq 4$

3. Find the value of x, y,w and z:

$$\begin{vmatrix} 2+x & 1 \\ 3-y & z \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = \begin{vmatrix} w & 7 \\ 8 & 5 \end{vmatrix}$$

4. Given A and B such that $P(A) = 4, P(B) = 2 \; and \; P(A \cap B) = 2$, find the value of P (B I A) and P(A I B)

5. Find the continuity of the function $f(x) = \begin{matrix} 3x-3; x \geq 2 \\ x+1; x \leq 2 \end{matrix}$

6. Find the rate of change of the area of the cube per second with respect to its edges when S = $5\sqrt{2}$

7. Integrate $\int \frac{x^2}{2} + \sqrt{x}$

8. Find the general equation $\frac{dy}{dx} = \frac{3x^2}{5y^4}$

9. Find the vector joining the points P(2, -3, 4) to Q(1, 6, -9) directed from P to Q.

10. Find the direction cosines of the line passing through the two points (1, 2, – 3) and (3, -5, 2)

GROUP-B

Total Marks: 60

11. Answer any 3 questions **$3 \times 4 = 12$**

a) Find the value of A and B

i) (a – 2b , 2a – b) = (8, 10)

ii) (3a – 2b, 6) = (3, 2a)

b) Prove that $cos^{-1}\left(\frac{20}{\sqrt{929}}\right) + tan^{-1}\left(\frac{2}{3}\right) + tan^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

c) Maximize and Minimize Z = 3x + 4y, Subject to $x + 2y \geq 100;\ 10x - 5y \leq 0;\ 2x + y \geq 100$

d) A table is given on food nutrition and what is the minimum cost of the supplement should be given with respect to the given constraints

	Vitamin D (unit/kg)	Vitamin K (unit/kg)	Cost(Rs./kg)
Food X	1	5	50
Food Y	3	2	100
The Required Measure of the Supplement.	6	12	Z = 50x + 100y

e) If A = $\begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix}$ and B = $\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$. Then find the value of $AB + A^2 + B^2$

12. Answer any 3 questions **$3 \times 4 = 12$**

a) Find the value of $X^2 + Y^2$; if $X + Y = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -2 & 2 \\ -2 & 1 & 1 \end{vmatrix}$ and XY = $\begin{vmatrix} 8 & 4 & 2 \\ -4 & 8 & 4 \\ -2 & -4 & 8 \end{vmatrix}$

b) Two Orchard Owner Ram and Shyam grow only three types of Oranges namely Blood Orange, Bitter Yellow and Sour Red. The sale (in Rupees) of these types of Oranges by both the orchard owners in the year of 2009 and 2010 are given in terms of matrix A and B.

I) What amount is gathered in the 2 years total by each orchard owner?

II) Which orchard owner has the highest profit and on which type of orange from 2009 to 2010.

III) Which orange was least popular and which owner owned it.

Sale in Year 2009 (in Rupees)

$$A = \begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 10000 & 12000 & 11500 \\ 3000 & 20000 & 10000 \end{vmatrix} \qquad \begin{matrix} Ram \\ Shyam \end{matrix}$$

Sale in Year 2010 (in Rupees)

$$B = \begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 15000 & 2000 & 15000 \\ 5000 & 15000 & 9000 \end{vmatrix} \quad \begin{matrix} Ram \\ Shyam \end{matrix}$$

c) Using elementary operation find inverse of the matrix

$$A = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix}$$

d) 2 dices are thrown and the number of their result is noted

I) The probability that the sum is more than 9

II) The probability that the product is more than 10 and less than that of 20

III) The probability that the sum is divisible by 5

e) A card is drawn from the deck of a well shuffled 52 cards

I) A red king card

II) A black queen card

III) A red 10 number card

13. Answer any 3 questions **$3 \times 4 = 12$**

a) Check whether $f(x) = \begin{matrix} 3x^2 + 5x; x > 0 \\ \frac{3}{4}x - \frac{9}{8}; x < 0 \end{matrix}$ is continuous or not.

b) Find the derivative of 2tan(4x + 9)

c) Find $\frac{dy}{dx}$; if $\frac{2}{3}y + 2cosy = 3sinx$

d) Find the equation of all lines having slope 3 and tangent at $2y + \frac{6}{5-x} = 0$

e) Integrate $\int sin(2x+3)cos(3x+2)dx$

14. Answer any 3 questions **$3 \times 4 = 12$**

a) Integrate $\int 2(x^2 + 5x - 24) + 3(5x+6)dx$

b) Solve the integral $\sqrt{(x^2 - 10x + 25)}$ dx

c) Solve the integral $\frac{1}{\sqrt{(6x^2-9x)}}dx$

d) Form the differential equation of the family of circle touching at x = 2 on x axis.

e) Find the general equation of $\frac{dy}{dx} = (x^2+1)(2y+3)$.

15. Answer any 3 questions **$3 \times 4 = 12$**

a) Solve $\frac{dy}{dx} + ysinx = \frac{cos^2x}{3}$

b) The angle formed between two planes in a Cartesian form with planes being

$\frac{2}{3}x + \frac{11}{3}y + \frac{7}{3}z + 12 = 0$ and $\frac{2}{3}x + \frac{1}{3}y + kz + 27 = 0$ is 45° . Find the value of k.

c) Find the value of ((a – b)(a + b) + (a + b) - (a – b)) if a = $i + 2j - 8k$ and b = $3i - 3j + 6k$

d) Find the unit vector perpendicular to (2a + 3b) and (3a – b) where $a = 2i + 3j - 6k$ & $b = 3i + 2j - 3k$

e) Find the Area of the rectangle with length l = $\frac{1}{2}i + \frac{1}{3}j + \frac{1}{3}k$ and breadth b = $\frac{2}{3}i + \frac{2}{3}j + \frac{2}{3}k$

GROUP-C

Total Marks: 30

16. Answer any 1 question **$6 \times 1 = 6$**

a) Prove

$$\frac{\frac{\pi}{4}+\tan^{-1}(\tan(\cos^{-1}\left(\frac{3}{5}\right))+\sin(\cos^{-1}\left(\frac{3}{5}\right))}{\frac{\pi}{4}+\tan^{-1}(\cos(\tan^{-1}\left(\frac{8}{6}\right))+\sin(\cos^{-1}\left(\frac{6}{10}\right))}=1$$

b) A pastry chef makes 3 delicacy for his opening night now due to highly decorated renovation he is little tight on budjet but the chef has a reputation of making outstanding sweet dish no matter whatever or how much quantity the ingredient is So he decided on 2 dishes on a single plate, Black Choco Pie & Milk Caramel Crossiant .Now 2 common ingredients in the dishes are Chocolate and Milk, the list of ingredients are given below. Find the minimum cost required to serve both the dish on a single plate.

	Chocolate (unit/kg)	Milk (unit/kg)	Cost(Rs./kg)
Black Choco Pie	10	50	500
Milk Caramel Crossiant	20	30	1000
The Required Measure of the Dish	200	200	Z = 500x + 1000y

17. Answer any 1 question: $\mathbf{6 \times 1 = 6}$

a) Solve equation below using matrix method

$4x-2y+z=8$

$3x+6y-4z=10$

$2x-5y+z=8$

b) While swimming in an ocean along Banda Beach (infamous for Shark Attacks) the chances of shark attacks 100m away from the shore are 80%. Now if swimming enthusiasts swim around 60m from the shore with grey and black swimsuit the chances of shark attacks reduce to 30% and 30m from the shore in lifeguard supervision reduces chances to 25%, Now to save one self the swimmer can choose either course. So find the probability that the swimmer chooses to swim around 60m from the shore with grey and black swimsuit.

18. Answer any 1 question: $\mathbf{6 \times 1 = 6}$

a) Find f ′(x) if $f(x) = (2cos\ x)^{sinx}$ for all 0 < x < π

b) A supernova explodes in the outer space near the Cassia Nova Star System forming a sphere of Dark and X- Energy at a rate of $200000\ km^3\ /\ sec$. How quickly will it hit its nearest Star names Copernicus if it's surface area is increasing, with its edges now at 10000 km

19. Answer any 1 question: $\mathbf{6 \times 1 = 6}$

a) Integrate $\frac{cos(\sqrt{x}+5)\times e^{\sqrt{x}}}{\sqrt{x}}dx$

b) Find the area of the region enclosed between the two circles: $(x-4)^2 + y^2 = 8$ and $x^2 + y^2 = 8$

20. Answer any 1 question: $\mathbf{6 \times 1 = 6}$

a) Find the angle and unit vector of two vectors given a = $2i - 7j + 4k$ and b = $4i + 3j - 2k$

b) Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\acute{\imath} + 3\acute{\jmath} + 9\acute{k}\right) = 12$ and $\vec{r} = \left(5\acute{\imath} + 5\acute{\jmath} + 5\acute{k}\right) = 10$ passing through (3, 3, 3)

Answers & Explanations

GROUP-A

1. Solution:

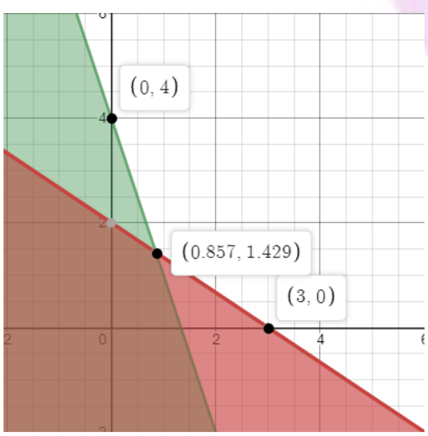
Given that $f(x) \; = \frac{3}{\sqrt{[x]^2 - 3[x] - 10}}$ f is defined by $[x]^2 - 3[x] - 10 > 0$

$([x]-5)([x]+2) > 0$ to this we get $[x] - 5 > 0$ and $[x] + 2 < 0$

Hence $x > 5, x < -2$ and therefore, the domain is $(-\infty, -2) \cup [4, \infty)$.

2. Solution:

To get a feasible solution by using constraints like $2x + 3y \leq 6$ and $3x + y \leq 4$ we get:



The corner points for the feasible regions are (0, 4), (0.857, 1.429), (3, 0), hence to find the maximum value we put the values in $4x \; + \; 5y \; = \; z$ we get:

Corner Point	Z = 4x + 5y	
0,4	20	maximum
0.857, 1.429	10.5	
3,0	12	

Therefore, the maximum value of Z = 4x + 5y is at (0, 4).

3. Solution:

Multiplying the matrices and comparing with rest

$$\begin{vmatrix} 3x+7 & x+5 \\ z-3y+9 & 3z-y+3 \end{vmatrix} = \begin{vmatrix} w & 7 \\ 8 & 5 \end{vmatrix}$$

After equating the matrices we get

$(z-3y+9=8)$
$(3z-y+3=5)$

After equating the equation, we find the value of z and y:

$3(z-3y+9=8)$
$1(3z-y+3=5)$

$$\begin{array}{r} 3z-9y+27=24 \\ 3z-y+3=5 \\ - \quad + \quad - \quad - \end{array}$$

$-8y+24=19$

$-8y=-5\,;\, y = 5/8$

Putting the value of y in $3z - 9y = -3$, we get the value of z $3z-9.\frac{5}{8}=-3;$ $\quad z=2.208$

$x+5=7, x=7-5$

$x=2$

Putting the value of x in $3x+7=w,\ 3(2)+7=w$

$6+7=w;\ \ 13=w$

Hence, the value of x, y, w and z are 2, 0.625, 13, 2.208.

4. Solution:

We have P (A I B) = $\frac{P(A \cap B)}{P(A)}$, using the value we get P (A I B) = $\frac{2}{4}$ and to find the value of P (B I A) we use the formula P (A I B) = $\frac{P(A \cap B)}{P(B)} = \frac{2}{2} = 1$

Hence, the value of P (A I B) and P (B I A) = 0.5 and 1.

5. Solution:

The function f(x) is defined as on the real line, hence,

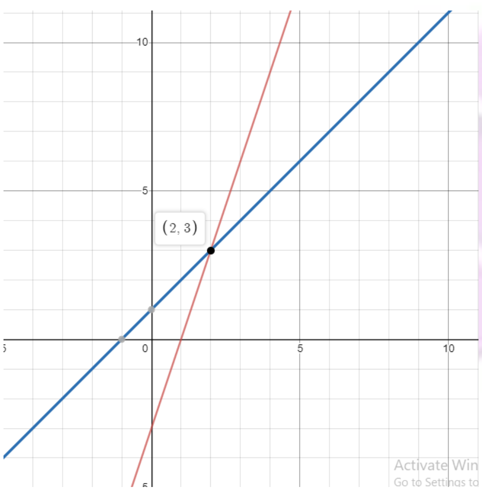
Case 1: $c \geq 2$, putting the value of c in $3x - 3$, we get $\lim_{x \to c}(3x - 3) = 3c - 3$

Thus f is continuous for all $x > 2$, as we put x = 2 we get $3x - 3$; $6 - 3 = 3$

Case 2: $c \leq 2$, putting the value of c in $x + 1$, we get, $\lim_{x \to c}(x + 1) = c + 1$

Thus f is continuous for all $x < 2$, as we put x = 2 we get $x + 1$; $2 + 1 = 3$

As we can see that the value on both the cases is 3, therefore the function f(X) is continuous as given in the figure below:



6. Solution:

The area of the cube is given by Area = S^2, therefore, the rate of change of area A with respect to side S is given by $\frac{dA}{dS} = \frac{d}{dS}(s^2) = 2S$. Hence, if the value of S = $5\sqrt{2}$, then $\frac{dA}{dS} = 10\sqrt{2}$;

Therefore,

The rate of change of the Area of the Cube is $10\sqrt{2}\ cm^2$.

7. Solution:

Simplify the integrate into parts we get:

$$\frac{1}{2}\int x^2 dx + \int \sqrt{x} dx$$

Integrating the first part we get:

$$\frac{1}{2}\int x^2 dx = \frac{1}{2}.\frac{x^3}{3}$$

Integrating the second part we get:

$$\int \sqrt{x}\, dx = \frac{2}{3}x^{\frac{2}{3}}$$

Joining both the parts

$$\frac{1}{2}\int x^2 dx + \int \sqrt{x} dx \quad = \frac{1}{2}.\frac{x^3}{3} + \frac{2}{3}x^{\frac{2}{3}} + C$$

Simplifying the answer from

$\frac{1}{2}.\frac{x^3}{3} + \frac{2}{3}x^{\frac{2}{3}} + C$ into $\frac{x^3}{6} + \frac{2}{3}x^{\frac{2}{3}} + C$

8. Solution:

To find the solution we interchange the position of x and y variables to get $\int 5y^4\, dy = \int 3x^2\, dx$

Finding the integrals, we get $\frac{5}{5}y^5 = \frac{3}{3}x^3 + C$

Simplifying the solution we get $y^5 = x^3 + C; \quad y = (x^3 + C)^{\frac{1}{5}}$

Hence, the differential solution is $y = (x^3 + C)^{\frac{1}{5}}$

9. Solution:

Since the vector is directed from P to Q it clearly states that P is the initial point and Q is the final point, the vector we need to find the points joint P and Q is $\overrightarrow{PQ}$

$$\overrightarrow{PQ} = (1-2)i + (6—3)j + (-9-4)k$$

Hence, the vector joining the points PQ is $\overrightarrow{PQ} = (-1)i + (9)j + (-13)k$.

10. Solution:

As we know that the direction cosine of the line is passing through A (a, b, c) and B (x, y, z) are given by:

$$\frac{x-a}{PQ} = \frac{y-b}{PQ} = \frac{z-c}{PQ}$$

Where PQ = $\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$

Putting the value of P and Q, we get:

PQ = $\sqrt{(3-1)^2 + (-5-2)^2 + (2-(-3))^2}$; $\sqrt{2^2 + (-7)^2 + 5^2} = \sqrt{78}$

Hence, to find the direction cosines between 2 points is

$$\frac{x-a}{PQ}, \frac{y-b}{PQ}, \frac{z-c}{PQ} \quad = \quad \frac{2}{\sqrt{78}}, \frac{-7}{\sqrt{78}}, \frac{5}{\sqrt{78}}$$

GROUP-B

11.

a) Solution:

Since ($a - 2b$, $2a - b$) = (8, 10) so,

$a - 2b = 8$ & $2a - b = 10$

Multiplying $a - 2b = 8$ with 2 and multiplying 1 with $2a - b = 10$ we get

$2(a - 2b = 8)$
$1(2a - b = 10)$

$2a - 4b = 16$
$2a - b = 10$

Subtracting the equations, we get

$-3b = 6; b = -2$

Putting the value of b in $2a - b = 10$, we get:

$2a - (-2) = 10; 2a = 12; \ a = 6$

Hence the value of a and b is 6 and -2 respectively. Answer

ii) Since (3a – 2b, 6) = (3, 2a), so

3a – 2b = 3 and 6 = 2a

Hence the value of a = $\frac{6}{2} = 3$

Putting the value of a in 3a – 2b = 3, we get: $3 \times 3 - 2b = 3$ or 6 = 2b

The value of b = 3

Hence, the value of a and b = 3 respectively.

b) Solution:

First, let us solve the part $\left[tan^{-1}\left(\frac{2}{3}\right) + tan^{-1}\left(\frac{1}{2}\right)\right]$

Using the formula, $tan^{-1}(x) + tan^{-1}(y) = tan^{-1}\left(\frac{x+y}{1+xy}\right)$ we get:

$$tan^{-1}\left(\frac{2}{3}\right) + tan^{-1}\left(\frac{1}{2}\right) = tan^{-1}\left(\frac{\frac{2}{3}+\frac{1}{2}}{1+\frac{2}{3}\cdot\frac{1}{2}}\right)$$

Simplifying the solution, we get

$$tan^{-1}\left(\frac{7}{8}\right)$$

Now as for $cos^{-1}\left(\frac{20}{\sqrt{929}}\right)$, convert it into tan, in tan we get $tan^{-1}\left(\frac{23}{20}\right)$, putting the $tan^{-1}\left(\frac{23}{20}\right)$ in place of $cos^{-1}\left(\frac{20}{\sqrt{929}}\right)$ we get

We get $tan^{-1}\left(\frac{23}{20}\right) + tan^{-1}\left(\frac{7}{8}\right) = \frac{\pi}{2}$

Again using the formula, $tan^{-1}(x) + tan^{-1}(y) = tan^{-1}\left(\frac{x+y}{1+xy}\right)$ we get:

$$tan^{-1}\left(\frac{\frac{23}{20}+\frac{7}{8}}{1+\frac{23}{20}\cdot\frac{7}{8}}\right) = tan^{-1}\left(\frac{108}{107}\right) = tan^{-1}(1.009) = \theta$$

Therefore, solving $tan^{-1}(1.009) = \theta$ we find the value of $\theta = 90° \; or \; \pi/2$.

Hence, Proved.

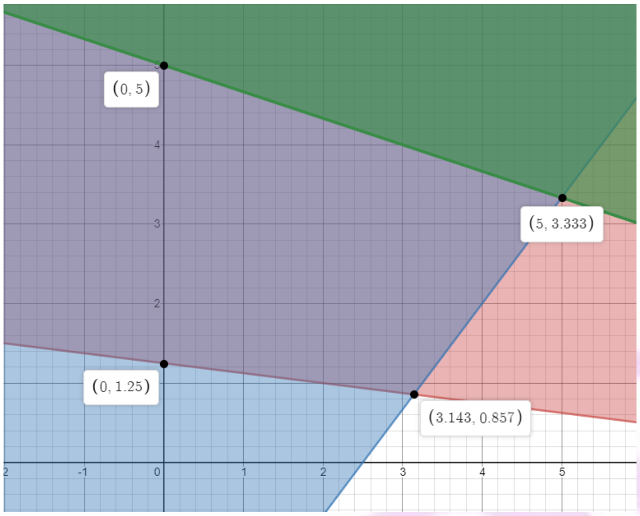
c) Solution:

The feasible region that defines the constraints is:

The corner of the feasible points is $(0,5), (0,1.25), (3.143,0.857), (5, 3.33)$

Putting the value in Z = 3x + 4y, we find:

Corner Points	Z = 3x + 4y	
(0,5)	20	
(0,1.25)	5	minimize
(3.143,0.857)	12.857	
(5, 3.33)	28.32	maximize



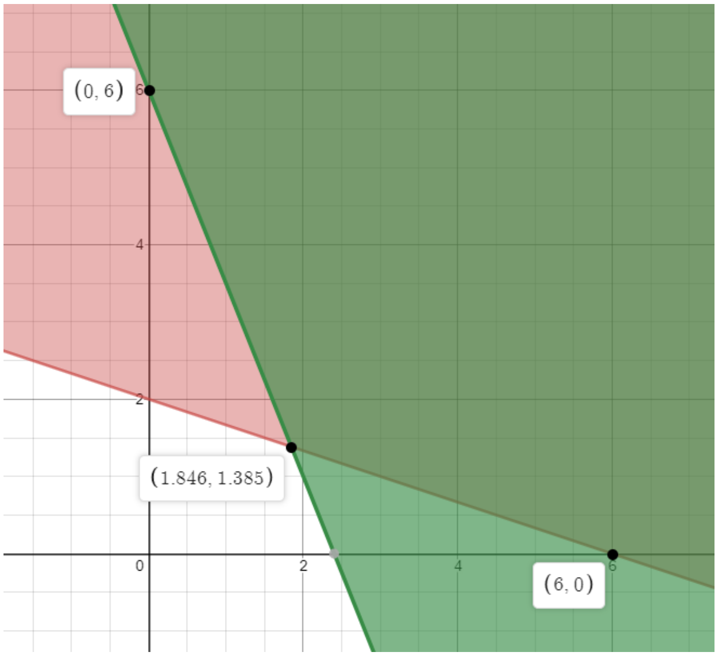
The maximum value of Z is in point (5, 3.333) and the minimum value is at (0, 1.25).

d) Solution:

The required constraints are $1X + 3Y \geq 6; \quad 5X + 2Y \geq 12$

Now to find a food supplement that is of minimum cost and contains 6 units of Vit. D and 12 units of Vit. K

Now the feasible region determine by the graph



Corner Points	Z = 50x + 100y	
0,6	600	
1.846, 1.358	227	Minimum
6, 0	300	

Acccording to the graph of feasibility we can see that point (1.846, 1.358) is the point where the cost will be minimum if the The Vitamin D and K has to be 6 and 12 units respectively.

e) Solution:

First let us find the product of AB that is

$\begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 7 & 8 \end{vmatrix}$, now let us find the square of A and B, we get

$$A^2 = \begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} \begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} = \begin{vmatrix} 0 & -5 \\ -5 & 0 \end{vmatrix}$$

$$B^2 = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = \begin{vmatrix} -3 & 0 \\ 0 & -3 \end{vmatrix}$$

To find the value of $AB + A^2 + B^2 = \begin{vmatrix} 8 & 7 \\ 7 & 8 \end{vmatrix} + \begin{vmatrix} 0 & -5 \\ -5 & 0 \end{vmatrix} + \begin{vmatrix} -3 & 0 \\ 0 & -3 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix}$.

Hence, the value of $AB + A^2 + B^2 = \begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix}$.

12.

a) Solution:

As the question says $X + Y = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -2 & 2 \\ -2 & 1 & 1 \end{vmatrix}$ and XY = $\begin{vmatrix} 8 & 4 & 2 \\ -4 & 8 & 4 \\ -2 & -4 & 8 \end{vmatrix}$ and find the value of $X^2 + Y^2$; we can use the formula of $(X+Y)^2 = X^2 + Y^2 + 2XY$

So putting the value in $(X+Y)^2 = X^2 + Y^2 + 2XY$

$$\left(\begin{vmatrix} 1 & 2 & -1 \\ 2 & -2 & 2 \\ -2 & 1 & 1 \end{vmatrix}\right)^2 = X^2 + Y^2 + 2\begin{vmatrix} 8 & 4 & 2 \\ -4 & 8 & 4 \\ -2 & -4 & 8 \end{vmatrix}$$

Therefore to find the value of $X^2 + Y^2$

We first find the value of $(X+Y)^2 = \left(\begin{vmatrix} 1 & 2 & -1 \\ 2 & -2 & 2 \\ -2 & 1 & 1 \end{vmatrix}\right)^2 = \begin{vmatrix} 7 & -3 & 2 \\ -6 & 10 & -4 \\ -2 & -5 & 5 \end{vmatrix}$

$\begin{vmatrix} 7 & -3 & 2 \\ -6 & 10 & -4 \\ -2 & -5 & 5 \end{vmatrix} = X^2 + Y^2 + \begin{vmatrix} 16 & 8 & 4 \\ -8 & 16 & 8 \\ -4 & -8 & 16 \end{vmatrix}$ multiplying 2 with $\begin{vmatrix} 8 & 4 & 2 \\ -4 & 8 & 4 \\ -2 & -4 & 8 \end{vmatrix}$

Hence, the value of $X^2 + Y^2 = \begin{vmatrix} -9 & -11 & -2 \\ 2 & -6 & -12 \\ 2 & 3 & -11 \end{vmatrix}$

b) Solution:

The total sale of year 2009 and 2010 is given by

$$A + B = \begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 10000 & 12000 & 11000 \\ 3000 & 20000 & 10000 \end{vmatrix} + \begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 15000 & 2000 & 15000 \\ 5000 & 15000 & 9000 \end{vmatrix} =$$

$$\begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 25000 & 14000 & 26000 \\ 8000 & 35000 & 19000 \end{vmatrix}$$

To find the highest profit and the type of orange that is grown from 2009 to 2010.

B – A =

$$\begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 15000 & 2000 & 15000 \\ 5000 & 15000 & 9000 \end{vmatrix} - \begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 10000 & 12000 & 11000 \\ 3000 & 20000 & 10000 \end{vmatrix} =$$

$$\begin{vmatrix} \text{Blood Orange} & \text{Bitter Yellow} & \text{Sour Red} \\ 5000 & -10000 & 4000 \\ 2000 & -5000 & -1000 \end{vmatrix}$$

According to the matrix it can be found out that the highest profit was grossed by Ram in 2010 on his Blood Oranges a total of Rs. 5000.

The least popular orange was the Bitter Yellow as both the Orchard Owner show poor sales record with Ram' sale being the Poorer of the two.

Hence, the least popular Orange was Bitter Yellow and the owner was Ram.

c) Solution:

Write A = IA ; I = $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} A$$

Using triangle law:

$$C_{1,1} = -1^{1+1} \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 1$$

$$C_{1,2} = -1^{1+2} \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 4$$

$$C_{1,3} = -1^{1+3} \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = -2$$

$$C_{2,1} = -1^{1+1}\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = -2$$

$$C_{2,2} = -1^{2+2}\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 1$$

$$C_{2,3} = -1^{2+3}\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 4$$

$$C_{3,1} = -1^{3+1}\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 4$$

$$C_{3,2} = -1^{3+1}\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = -2$$

$$C_{3,3} = -1^{3+1}\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 1$$

$$C^T = \begin{vmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 1(1) + 2(4) - 0(-2) = 9$$

$$A^{-1} = \frac{1}{|A|}C^T = \frac{1}{9}\begin{vmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{vmatrix} = \begin{vmatrix} \frac{1}{9} & -\frac{2}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{1}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{4}{9} & \frac{1}{9} \end{vmatrix}$$

Therefore, the inverse of A is $\begin{vmatrix} \frac{1}{9} & -\frac{2}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{1}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{4}{9} & \frac{1}{9} \end{vmatrix}$

d) Solution:

The total sample space of the dices rolled is 6^6 = 36

The total number of possiblities when dices are rolled is

$$(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$$
$$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$$
$$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$$
$$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$$
$$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$$
$$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)$$

I) The probability that the sum is more than 9

Now the sum of dices rolled that can be more than 9 is $(4,6),(5,4),(5,5),(5,6),(6,4)(6,5)(6,6) = \frac{7}{36}$. Answer

II) The probability that the product is more than 10 and less than that of 20

The number of possibilites are $(2,6),(3,4),(3,5),(3,6),(4,3),(4,4),(5,3),(6,2),(6,3) = \frac{9}{36}$. Answer

III) The probability that the sum is divisible by 5

The number of possibilities are $(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4) = \frac{7}{36}$.

e) Solution:

I) A red king card

Total number of possibilites or total number of cards = 52

There are 4 types of card and 26 of them are black and 26 red, out of which 4 king of which 2 are black and 2 are red, hence possibilities of red king card = 2

$The\ probability\ of\ a\ Red\ King\ Card\ = \frac{Number\ of\ red\ king}{Total\ number\ of\ cards} = \frac{2}{52}$. Answer

II) A black queen card

Total number of possibilites or total number of cards = 52

There are 4 types of card and 26 of them are black and 26 red, out of which 4 queen of which 2 are black and 2 are red, hence possibilities of black queen card = 2

$The\ probability\ of\ a\ Black\ Queen\ Card\ = \frac{Number\ of\ black\ queen}{Total\ number\ of\ cards} = \frac{2}{52}$ .Answer

III) A Red number 10 card

Total number of possibilites or total number of cards = 52

There are 4 types of card and 26 of them are black and 26 red, out of which 4 10's of which 2 are black and 2 are red, hence possibilities of red number 10 card = 2

$$\textit{The probability of a } \text{red number 10 card} = \frac{\text{Red number 10 card}}{\textit{Total number of cards}} = \frac{2}{52}$$

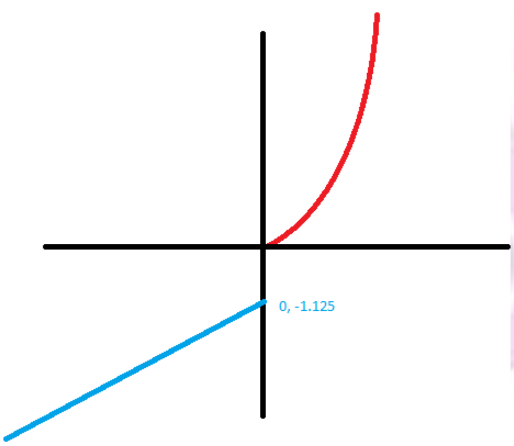
13.

a) Solution:

To check the continuity of the function f let us first check if $f(c)$ exists or not, putting 0 in $f(x) = \begin{matrix} 3x^2 + 5x; x > 0 \\ \frac{3}{4}x - \frac{9}{8}; x < 0 \end{matrix}$ we get $f(x) = 3x^2 + 5x;\ f(c) = 3c^2 + 5c$ with c = 0 we get: $f(0) = 0$

Now checking in $f(x) = \frac{3}{4}x - \frac{9}{8}; f(c) = \frac{3}{4}c - \frac{9}{8}$ with c = 0; we get $f(0) = 0 - \frac{9}{8}; f(0) = -\frac{9}{8}$

Therefore the graph for $f(x) = 3x^2 + 5x$ and $f(x) = \frac{3}{4}x - \frac{9}{8}$ is



Hence as we can see that RHS $\lim_{x\to 0}(3x^2 + 5x)$, we get $\lim_{x\to 0}(3x^2 + 5x) = 0$ and LHS $\lim_{x\to 0}\left(\frac{3}{4}x - \frac{9}{8}\right) = \frac{9}{8}$. Therefore as we can see that LHS ≠ RHS. The fuction f(x) is discontinous at 0.

b) Solution:

Let f(x) = 2tan(4x + 9), u(x) = 4x + 9 and v(t) = 2tan t. Then

(v o u)(x) = v(u(x)) = v(2x + 3) = 2tan(4x + 9).

Thus f is a composite function. Putting t = u(x),

we get $\frac{dv}{dt} = 2sec^2 t$ and $\frac{dt}{dx} = 4$

Hence by using chain rule we get $\frac{df}{dx} = \frac{dv}{dt}.\frac{dt}{dx} = 8\sec^2(4x + 9)$

Therefore, the derivative of 2tan(4x + 9) is $8\sec^2(4x + 9)$.

c) Solution:

Differentiating the entire equation we get;

$\frac{2}{3}\frac{dy}{dx} \quad + 2cosy\ \frac{dy}{dx} = \frac{3dsinx}{dx}$

Taking $\frac{dy}{dx}$ common we get

$\frac{dy}{dx}\left(\frac{2}{3} \quad + 2cosy\right) = \frac{3dsinx}{dx}$

Simplifying the equation in

$\frac{dy}{dx} \ = \frac{3cosx}{\left(\frac{2}{3}\ \ +2cosy\right)} = \frac{9cosx}{2(1+3cosy)}$.

d) Solution:

The slope of tangent is defined by $\frac{dy}{dx}$ i.e. 3

Now we have to differentiate the equation with respect to x; i.e. $\frac{2dy}{dx} = \frac{d}{dx}.\frac{6}{5-x}$

$\frac{2dy}{dx} = \frac{6}{(5-x)^2}$, putting the value of $\frac{dy}{dx}\ =\ 3$,

we get $2 \times 3 = \ \frac{6}{(5-x)^2}$ i.e. $\frac{6}{6} = (5 - x)^2$; $\quad 1 = (5 - x)^2$

now solving for x we get:

$x = 6, 4$

Putting x as 6 and 4 we get y = 3, -3

Putting the values of x and y to get the equation of lines

For (6, 3) and (4, -3) we get

$y - 3 = 3(x - 6)$ putting the value of x =6 and y = 3 in $y_2 - y_1 = 3\ (x_2 - x_1)$

$y + 3 = 3(x - 4)$putting the value of x =4 and y = -3 in $y_2 - y_1 = 3\ (x_2 - x_1)$

Hence there are 2 line equations $y - 3 = 3(x - 6)$ and $y + 3 = 3(x - 4)$.

e) Solution:

Using the formula $sin(X)cos(Y) = \frac{1}{2}\left(sin(y+x) - sin(y-x)\right), cosxsinx = \frac{1}{2}sin(2x)$

$$\frac{1}{2}\int sin(5x+5)dx - \frac{1}{2}\int sin(x-1)dx$$

Now solving

$$\frac{1}{2}\int sin(5x+5)dx$$

Taking $\sin(5x+5) = u; dx = \frac{1}{5}du$

$$\frac{1}{5}\int sin(u)du$$

$$\int sin(u)du = -cos\,(u)$$

Putting $u = 5x + 5; we\ get - \frac{cos(5x+5)}{5}$

Now solving $for \quad \frac{1}{2}\int sin(x-1)dx$

Solving this part similarly like the above we get

$-cos(x-1)$

$$\frac{1}{2}\int sin(5x+5)dx - \frac{1}{2}\int sin(x-1)dx = \frac{1}{2}\left(-\frac{cos(5x+5)}{5} - cos(x-1)\right) + C$$

we get $\left(-\frac{cos(5x+5)}{10}\ \frac{-cos(x-1)}{2}\right) + C.$

14.

a) Solution:

To solve first simplify and then integrate it in parts like $2\int x^2dx + 25\int xdx - 30\int 1dx,$

Now let us solve the parts

$$2\int x^2dx = 2.\frac{x^3}{3}$$

Solving the second part

$$25\int xdx = \frac{25x^2}{2}$$

Solving the third part

$$30\int 1dx = 30x$$

now plugging the solved integrals we get:

$$2\int x^2dx + 25\int xdx - 30\int 1dx \quad = \frac{2}{3}x^2 + \frac{25}{2}x^2 - 30x + C$$

$$\frac{x}{6}(4x^2 + 75x - 180) + C.$$

b) Solution:

solving the quadratic under the square root we get $(x-5)^2$

now solving the integral we get $(x-5)^{2\frac{1}{2}} = (x-5)$

Solving the integrals we get

$$\int xdx - 5\int 1dx$$

first solve the $\int xdx = \frac{x^2}{2}$ after this solve

$$5\int 1dx = 5x$$

Putting both of them together we get

$$\frac{x^2}{2} - 5x = \frac{x^2 - 10x}{2}$$

Therefore, the solution is $\frac{x^2-10x}{2}$.

c) Solution:

$$\int \frac{1}{\sqrt{3}\sqrt{x}\sqrt{2x-3}}dx$$

Taking $u = \frac{\sqrt{2}.\sqrt{x}}{\sqrt{3}}$ $dx = \sqrt{2}\sqrt{3}\sqrt{x}du$

$\int \frac{\sqrt{2}}{\sqrt{3u^2-3}}$ du Simplify the integral in terms of u

$$\frac{\sqrt{2}}{\sqrt{3}}\int \frac{1}{\sqrt{u^2-1}}\,\text{du}$$

Placing u = sec(v); du = sec(v)tan(v)dv

$$\int \frac{1}{\sqrt{u^2-1}}\,\text{du} = \frac{\sec(\text{v})\tan(\text{v})}{\sqrt{\sec^2(\text{v})-1}}\text{dv} = \int secvdv$$

Placing h = tan(v) + sec(v), we get

$$\int \frac{secvtanv + sec^2v}{tanv + secv} = \int \frac{1}{h} = log\ h$$

$$log\ h = log\big(tan(v) + sec(v)\big) = log\sqrt{(u^2-1)} + u$$

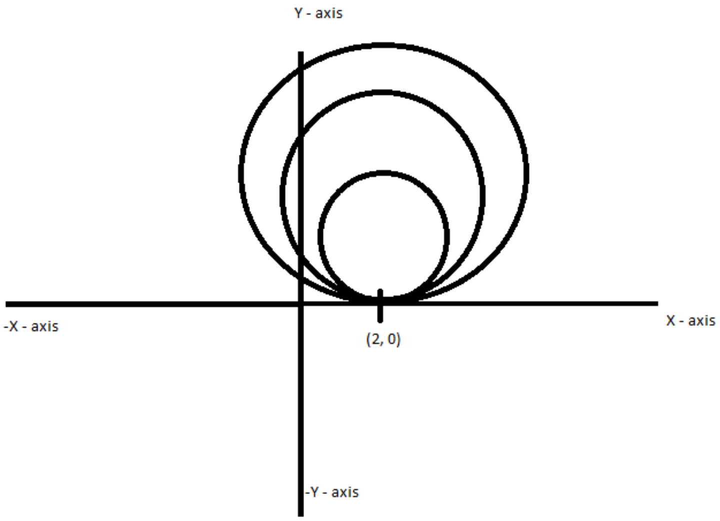
Put $u = \frac{\sqrt{2}.\sqrt{x}}{\sqrt{3}}$ back in the equation we get

$$\frac{\sqrt{2}log\left(\frac{\sqrt{2}.\sqrt{x}}{\sqrt{3}} + \sqrt{\frac{2x-3}{3}}\right)}{\sqrt{3}} = \frac{\sqrt{2}log\sqrt{2x-3} + \sqrt{2}.\sqrt{x}}{\sqrt{3}} + C.$$

Therefore the solution is $\frac{\sqrt{2}log\sqrt{2x-3}+\sqrt{2}.\sqrt{x}}{\sqrt{3}} + C$

d) Solution:

Let us denote the family of circle by C, which touches the x axis at 2. Let (2, a) be the center of the circle. Therefore the family of the circle be



Therefore the equation of family C = $(x-2)^2+(y-a)^2=a^2$; a is an arbitary element. Differentiating $(x-2)^2+(y-a)^2=a^2$ wrt x we get:

$$2x + 2y\frac{dy}{dx} = 2a\frac{dy}{dx}$$

Taking 2 common in the equation we get

$x + y\frac{dy}{dx} = a\frac{dy}{dx}$; $a = \frac{x+\frac{ydy}{dx}}{\frac{dy}{dx}}$; Submitting the value of $a = \frac{(x-2)^2+y^2}{2y}$ in $a = \frac{x+\frac{ydy}{dx}}{\frac{dy}{dx}}$ we get

$$\frac{(x-2)^2+y^2}{2y} = \frac{x+\frac{ydy}{dx}}{\frac{dy}{dx}}$$

Solving in terms of $\frac{dy}{dx}$ we get $\frac{(x-2)^2+y^2}{2y} = \frac{x+\frac{ydy}{dx}}{\frac{dy}{dx}}$

$$\frac{dy}{dx}\frac{(x-2)^2+y^2}{2y} - \frac{ydy}{dx} = x$$

$$\frac{dy}{dx}\left(\frac{(x-2)^2+y^2}{2y} - y\right) = x; \quad \frac{dy}{dx} = \frac{x}{\left(\frac{(x-2)^2+y^2}{2y} - y\right)} = \frac{x}{\left(\frac{(x-2)^2-y^2}{2y}\right)} = \frac{2yx}{(x-2)^2-y^2}$$

Hence, the family of equation of circle C is $\frac{2yx}{(x-2)^2-y^2} = \frac{dy}{dx}$

e) Solution:

Separate the term of x and y of $\frac{dy}{dx} = (x^2+1)(2y+3)$ we get:

$$\frac{dy}{2y+3} = (x^2+1)dx$$

Integrating both sides we get

$$\int \frac{dy}{2y+3} = \int (x^2+1)dx$$

Let us first integrate the y part

$$\int \frac{dy}{2y+3} = \frac{log(2y+3)}{2}$$

Now solving the x part

$$\int (x^2+1)dx = \frac{x^3}{3} + x + C$$

Equating both the integral we find the solution of $\frac{log(2y+3)}{2} = \frac{x^3}{3} + x + C$

$\frac{log(2y+3)}{2} - \frac{x^3}{3} - x = C.$

15.

a) Solution:

The diffrential equation is in form of $\frac{dy}{dx} + Py = Q$ where P and Q are constant and function of x, is also first order of linear differential equation.

Now let us find the integrating factor I.F. = $e^{\int Pdx}$

And let us write the differential equation in terms of $y\,(I.F) = \int (Q \times I.F)dx + C$

First solve $\int Pdx = \int sinx\, dx$

$$\int sinx\, dx = -cosx$$

Therefore the integrating factor is –cosx

Now put the I.F. in $y\,(I.F) = \int (Q \times I.F)dx + C$ we get;

$y\,(-cosx) = \int (Q \times -cosx)dx + C$ with Q = $\frac{cos^2x}{3}$

$$y\,(-cosx) = \int \left(\frac{cos^2 x}{3} \times -cosx\right) dx + C$$

Integrating $\int \left(\frac{cos^2 x}{3} \times -cosx\right) dx$ we get

$$\int \left(\frac{cos^2 x}{3} \times -cosx\right) dx = \int \left(-\frac{cos^3 x}{3}\right) dx$$

$$\int \left(-\frac{cos^3 x}{3}\right) dx = \frac{sin^3 x - 3sinx}{9} + C$$

Putting the solved equation in $y\,(-cosx) = \int \left(\frac{cos^2 x}{3} \times -cosx\right) dx + C$ we get

$$y = \frac{sin^3 x - 3sinx}{9(-cosx)} + \frac{C}{(-cosx)}.$$

b) Solution:

The angle between 2 planes in a Cartesian form with planes being $\frac{2}{3}x + \frac{11}{3}y + \frac{7}{3}z + 12 = 0$ and $\frac{2}{3}x + \frac{1}{3}y + kz + 27 = 0$ is 45.

The direction ratios of the normal to the planes are A1 , B1 , C1 and A2 , B2 , C2 respectively.

A1 , B1 , C1 = 2/3 , 11/3 , 7/3 and A2 , B2 , C2 = 2/3, 1/3, k

$$Cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$$

$$\cos 60 = \frac{\frac{2}{3}.\frac{2}{3} + \frac{11}{3}.\frac{1}{3} + \frac{7}{3}.k}{\sqrt{\frac{174}{9}}\sqrt{\frac{5}{9} + k^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{\frac{5}{3} + \frac{7k}{3}}{4.4\sqrt{\frac{5}{9} + k^2}}$$

$$4.4\sqrt{\frac{5}{9} + k^2} = \sqrt{2}\left(\frac{5}{3} + \frac{7k}{3}\right)$$

Squaring both sides we get

$$19.36.\left(\frac{5}{9} + k^2\right) = 2.\left(\frac{5}{3} + \frac{7k}{3}\right)^2$$

Simplying in terms of quadratic equation we get

$$\frac{14}{9}+\frac{70k}{9}+\frac{49}{9}k^2$$

Now solving for k we get

K = - 0. 24, -1.18.

c) Solution:

To find the solution of (a – b)(a + b) + (a + b) - (a – b) we use a^2-b^2+2b

Now let us find the value of $a^2=|i+2j-8k|=|(1)i+(4)j+(64)k|$

The value of $b^2=|3i-3j+6k|=|(9)i+(9)j+(36)k|$

The value of 2b = $2(3i-3j+6k)=6i-6j+12k$

Hence the value of $a^2-b^2+2b=\ (1)i+(4)j+(64)k-(\,(9)i+(9)j+(36)k)+6i-6j+12k=2i-11j+40k$

Hence, the value of (a – b)(a + b) + (a + b) - (a – b) = $2i-11j+40k$.

d) Solution:

We have $a=2i+3j-6k$ & $b=3i+2j-3k$

First let us find (2a + 3b) and (3a – b)

2a = $4i+6j-12k$; 3b = $9i+6j-9k$ and 3a = $6i+9j-18k$

Solving (2a + 3b) = $4i+6j-12k+\ (9i+6j-9k)=\ 13i+12j-21k$

(3a – b) = $(6i+9j-18k)-(3i+2j-3k)=3i+7j-15k$

Now to find which vector is perpendicular to both $13i+12j-21k$ & $3i+7j-15k$

$$(2a\ +\ 3b)\times\ (3a-\ b)=\begin{vmatrix} i & j & k \\ 13 & 12 & -21 \\ 3 & 7 & -15 \end{vmatrix}=-33i+132j+55k$$

C = $-33i+132j+55k$ and $|c|=\sqrt{(-33^2+132^2+55^2)}=\ \sqrt{21538}\approx 147$

Therefore the unit vector is $\frac{c}{|c|}=\ \frac{-33}{147}i+\frac{132}{147}j+\frac{55}{147}k$. Answer

e) Solution:

To find the area of a rectangle we use $l \times b = \left(\frac{1}{2}i + \frac{1}{3}j + \frac{1}{3}k\right) \times \left(\frac{2}{3}i + \frac{2}{3}j + \frac{2}{3}k\right)$

So the area of the rectangle is $|l \times b| = \begin{vmatrix} i & j & k \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \end{vmatrix}$

Therefore, $|l \times b| = -\frac{j}{9} + \frac{k}{9}$

Hence as $|l \times b| = \sqrt{|a^2 i + b^2 j + c^2 k|} = \sqrt{\left|\left(-\frac{1}{9}\right)^2 + \left(\frac{1}{9}\right)^2\right|}$

$$|l \times b| = \sqrt{\frac{2}{81}} = \frac{\sqrt{2}}{9} unit^2$$

Hence the area of the rectangle is $\frac{\sqrt{2}}{9}\ unit^2$.

GROUP-C

16.

a) Solution:

Let us first solve the above part i.e. $\frac{\pi}{4} + \tan^{-1}(\tan(\cos^{-1}\left(\frac{3}{5}\right)) + \sin(\cos^{-1}\left(\frac{3}{5}\right))$

Now to convert $\cos^{-1}\left(\frac{3}{5}\right)$ into sin and tan we get

$\theta = \cos^{-1}\left(\frac{3}{5}\right)$; $cos\theta = \frac{3}{5}$ Therefore, as formula says $cos\theta = \frac{base}{hypotenuse}$ so the formula for tan and sin are

$$\theta = tan^{-1}\frac{height}{base} \;\; ; \;\; \theta = \sin^{-1}\frac{height}{hypotenuse}$$

Base= 3 and hypo= 5 so height= $\sqrt{hypo^2 - base^2}$; height= 4

So, $\theta = tan^{-1}\frac{4}{3}$; $\theta = \sin^{-1}\frac{4}{5}$

$$\frac{\pi}{4} + \tan^{-1}(\tan(tan^{-1}\frac{4}{3}) + \sin(\sin^{-1}\frac{4}{5}))$$

$\frac{\pi}{4} + \tan^{-1}\left(\frac{32}{15}\right)$ ____________________________________ 1

Now solving the denominator part we get by solving

$\frac{\pi}{4} + \tan^{-1}(\cos(\tan^{-1}\left(\frac{8}{6}\right)) + \sin(\cos^{-1}\left(\frac{6}{10}\right))$

Now to convert $\tan^{-1}\left(\frac{8}{6}\right)$ into sin and cos we get

$\theta = \tan^{-1}\left(\frac{8}{6}\right)$; $tan\theta = \left(\frac{8}{6}\right)$ Therefore, as formula says $tan\theta = \frac{height}{base}$ so the formula for cos and sin are

$\theta = cos^{-1}\frac{base}{hypo}$; $\theta = \sin^{-1}\frac{height}{hypo}$

Height = 8 and base = 6 so hypo= $\sqrt{height^2 + base^2}$; hypo= 4

So, $\theta = cos^{-1}\frac{6}{10}$; $\theta = \sin^{-1}\frac{8}{10}$

Putting $\theta = cos^{-1}\frac{6}{10}$; $\theta = \sin^{-1}\frac{8}{10}$ in
$\frac{\pi}{4} + \tan^{-1}(\cos(\tan^{-1}\left(\frac{8}{6}\right)) + \sin(\cos^{-1}\left(\frac{6}{10}\right))$ we get
$\frac{\pi}{4} + \tan^{-1}(\cos(cos^{-1}\frac{6}{10}) + \sin(\sin^{-1}\frac{8}{10})$

$\frac{\pi}{4} + \tan^{-1}(\frac{6}{10} + \frac{8}{10}) = \frac{\pi}{4} + \tan^{-1}\frac{14}{10}$ ______________________________ 2

Therefore, the question can be written as (2) divide by (1) that will be

$$\frac{\frac{\pi}{4} + \tan^{-1}\left(\frac{32}{15}\right)}{\frac{\pi}{4} + \tan^{-1}\left(\frac{14}{10}\right)}$$

$\frac{\pi}{4}$ can be written as $\tan^{-1} 1$, therefore,
$\frac{\frac{\pi}{4}+\tan^{-1}\left(\frac{32}{15}\right)}{\frac{\pi}{4}+\tan^{-1}\left(\frac{14}{10}\right)}$ can be written as $\frac{\left(\tan^{-1}1+tan^{-1}\left(\frac{32}{15}\right)\right)}{\left(\tan^{-1}1+tan^{-1}\left(\frac{14}{10}\right)\right)}$

Therefore according to the identity $\tan^{-1}x + \tan^{-1}y = \tan^{-1}(\frac{x+y}{1+xy})$ so

$$\frac{\left(\tan^{-1}1+tan^{-1}\left(\frac{32}{15}\right)\right)}{\left(\tan^{-1}1+tan^{-1}\left(\frac{14}{10}\right)\right)} = \frac{\tan^{-1}(\frac{x+y}{1+xy})}{\tan^{-1}(\frac{x+y}{1+xy})} = \frac{\tan^{-1}\left(\frac{1+\frac{32}{15}}{1+\frac{32}{15}}\right)}{\tan^{-1}\left(\frac{1+\frac{14}{10}}{1+\frac{14}{10}}\right)}$$

Further simplifying we get:

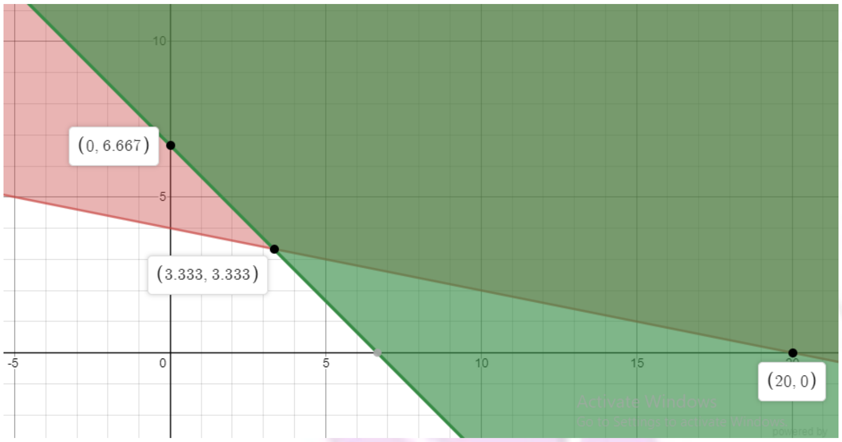
$$\frac{\tan^{-1}\left(\frac{1+\frac{32}{15}}{1+\frac{32}{15}}\right)}{\tan^{-1}\left(\frac{1+\frac{14}{10}}{1+\frac{14}{10}}\right)} = \frac{\tan^{-1}(1)}{\tan^{-1}(1)} = 1$$

Hence, Proved.

b) Solution:

The dishes are forming constraint in order of $10x + 20y \geq 200$, $50x + 30y \geq 200$ and $30x + 30y \geq 200$

To find minimum price to make the sweet dish is formed in a graph given below



Corner Points	Z = 500x + 1000y	
0,6.67	6670	
3.3333, 3.3333	4999	Minimum
20, 0	10000	

The minimum price for the ingrediant to make both the dish is (3.3333, 3.3333) Rs. 4999. Hence to make a single dish we have to put 3.333 kg of both Black Choco Pie or Milk Caramel Crossiant.

17.

a) Solution:

The system of equation can be written as AX = B

$A = \begin{vmatrix} 4 & -2 & 1 \\ 3 & 6 & -4 \\ 2 & -5 & 1 \end{vmatrix}$, X = $\begin{vmatrix} x \\ y \\ z \end{vmatrix}$, B = $\begin{vmatrix} 8 \\ 10 \\ 8 \end{vmatrix}$

Find the value of $|A|$

$|A| = -61$

As A exist in inverse let us find $A^{-1} = \begin{vmatrix} 14/61 & 3/61 & -2/61 \\ 11/61 & -2/61 & -19/61 \\ 27/61 & -16/61 & -30/61 \end{vmatrix}$

Therefore X = $A^{-1}B = \begin{vmatrix} 14/61 & 3/61 & -2/61 \\ 11/61 & -2/61 & -19/61 \\ 27/61 & -16/61 & -30/61 \end{vmatrix} \begin{vmatrix} 8 \\ 10 \\ 8 \end{vmatrix}$

Calculating the above matrix we get $\begin{vmatrix} 14/61 & 3/61 & -2/61 \\ 11/61 & -2/61 & -19/61 \\ 27/61 & -16/61 & -30/61 \end{vmatrix} \begin{vmatrix} 8 \\ 10 \\ 8 \end{vmatrix} = \begin{vmatrix} 20 \\ 52 \\ -26 \end{vmatrix}$

Therefore the value of x = 20; y = 52 and z = -26.

b) Solution:

Let the events be denoted by A, E, F with

A = the chances of shark attacks 100m away from the shore are 80%(0.80)

B = swimming enthusiasts swim around 60m from the shore with grey and black swimsuit the chances of shark attacks reduce to 30%(0.30)

C = swim around 30m from the shore in lifeguard supervision reduces chances to 25%(0.25)

P(A) = 0.80,

Now the swimmer can choose either or B or C;

P(B) = P(C) = 0.50

Now to find the probability that the swimmer chooses to swim around 60m from the shore with grey and black swimsuit

P(BIA) = $\frac{P(B)P(AIB)}{P(B).P(AIB)+P(C).P(AIC)}$

P(BIA) = $\frac{0.5.0.56}{0.5.0.56+0.5.0.6} = \frac{0.28}{0.58}$

Therefore, the probability that the swimmer chooses to swim around 60m from the shore with grey and black swimsuit = $\frac{0.28}{0.58}$.

18.

a) Solution:

The function y = $(2cos\ x)^{sinx}$ is applied for all positive real numbers. Taking logarithms, we get

$$log\ y = (2cos\ x)^{sinx} = sin\ x\ log\ 2cos\ x$$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{d}{dx}\ sin\ x\ log\ 2cos\ x$$

Now to find the differentiation of $\frac{d}{dx}$ sin x $log\ 2cos\ x$

$$\frac{d}{dx}[sin(x)] \cdot log\big(2cos(x)\big) + sin(x) \cdot \frac{d}{dx}\big[log\big(2cos(x)\big)\big]$$

$$cos(x).\ log\big(2cos(x)\big) + \frac{1}{2cos(x)} \cdot \frac{d}{dx}[2cos(x)] \cdot sin(x)$$

$$\cos(x)\log(2\cos(x)) + \frac{2 \cdot \frac{d}{dx}[\cos(x)] \cdot \sin(x)]}{2\cos(x)}$$

$$\cos(x)\log\big(2\cos(x)\big) - \frac{\sin^2 x}{\cos(x)}$$

$$\frac{dy}{dx} = y\left(\cos(x)\log\big(2\cos(x)\big) - \frac{\sin^2 x}{\cos(x)}\right)$$

Putting the value of y = $(2cos\ x)^{sinx}$ we get

$$(2cos\ x)^{sinx}\left(\cos(x)\log\big(2\cos(x)\big) - \frac{\sin^2 x}{\cos(x)}\right) = \frac{dy}{dx}.$$

b) Solution:

The rate at which the volume of the supernova explosion spreads is

$$\frac{dv}{dt} = 200\, {}^{km^3}/_{sec}$$

The volume of a sphere is $\left(\frac{4}{3}.\pi r^3\right)$, with r being the radius

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{4}{3}.\pi r^3\right)$$

$Since\ r\ is\ the\ variable\ hence\ r = x$

$$200000 = \frac{4}{3}\pi \int x^3$$

$$200000 = \frac{4}{3}\pi \frac{d}{dx}(x^3).\frac{dx}{dt}$$

$$200000 = \frac{4}{3}\pi\ 3x^2 .\frac{dx}{dt}$$

$$\frac{50000}{\pi x^2} = \frac{dx}{dt}$$

To calculate how quickly the explosion hits the nearest sun

$$\frac{ds}{dt} = \frac{d}{dt}\left(\frac{50000}{\pi x^2}\right)$$

the surface area of the explosion is 4πr^2, since r is variable r=x.

$$\frac{ds}{st} = \frac{d}{dx}(4\pi x^2)\frac{dx}{dt}$$

$$\frac{ds}{dt} = (4\pi 2x).\frac{50000}{\pi x^2}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{4 \times 2 \times 50000}{x} = \frac{400000}{x}$$

$if\ x = 10000km;$

$$\frac{ds}{dt} = \frac{400000}{10000} = 40km^2/sec.$$

Therefore, the speed at which the supernova explosion will hit the nearest sun is $40km^2/sec$.

19.

a) Solution:

Putting the value of u = $\sqrt{x}$; dx = $2\sqrt{x}$du

$$2\int e^u cos(u+5)du$$

$$\int e^u cos(u+5)du$$

Solving the equation in terms of $\int fg' = fg - \int f'g$

$$\mathrm{e^u cos(u+5) - \int - e^u sin(u+5)du}$$

$$\mathrm{e^u cos(u+5) - \left(-e^u sin(u+5) - \int -e^u cos(u+5)du\right)}$$

The integral $\int e^u cos(u+5)du$ appears again on the right side of the equation, we can solve for it:

$$\frac{(\mathrm{e^u\ sin(u+5)+e^u\ cos(u+5)})}{2}$$

Putting the value of u = $\sqrt{x}$ back we get

$$sin(\sqrt{x}+5)e^{\sqrt{x}} + cos(\sqrt{x}+5)e\sqrt{x} + C$$

Therefore, the solution of $\frac{cos(\sqrt{x}+5)\times e^{\sqrt{x}}}{\sqrt{x}}dx = sin(\sqrt{x}+5)e^{\sqrt{x}} + cos(\sqrt{x}+5)e\sqrt{x} + C$

b) Solution:

Equation of red circle is a circle with centre at the 4 and radius 2.828. Equation of blue circle is a circle with centre at origin and radius 2.828. Solving both circle equation we get

$$(x-4)^2 + y^2 = x^2 + y^2$$

$$x^2 + 16 - 8x + y^2 = x^2 + y^2$$

$$16 = 8x$$

$2 = x$ Putting x = 2 we get y as 2

Thus, the point of intersection is at (2, 2)

To find the area of the enclosed surface between 2 circle is

Area of first Circle + Area of the second Circle

$= 2\left[\int_0^{2.828} ydx + \int_{2.828}^{6.828} ydx\right]$

Putting the value of y we get

$= 2\left[\int_0^{2.828} \left(\sqrt{8-(x-4)^2}\right) dx + \int_{2.828}^{6.828} \sqrt{8-x^2}\, dx\right]$

Solving the integral, we get

$$\left[\frac{(x-4)\sqrt{-x^2+8x-8} - 8.sin^{-1}\left(\frac{8-2x}{2^{\frac{5}{2}}}\right)}{2}\right] - \left[\frac{x^2+16x}{2}\right] + C$$

$\left[\frac{(x-4)\sqrt{-x^2+8x-8} - 8.sin^{-1}\left(\frac{8-2x}{2^{\frac{5}{2}}}\right)}{2}\right]$ Putting x = 2 we get $\left[\frac{(2-4)4-8.sin^{-1}\left(\frac{8-4}{2^{\frac{5}{2}}}\right)}{2}\right] = -(4+\pi)$

Putting x = 1.172 we get $[-2.34 - 2.\pi]$

Subtracting $-(4+\pi) + [2.34 + 2.\pi] = \pi - 1.66$

Putting value 2 and 2.828 in $\left[\frac{x^2+16x}{2}\right]$ we get $\left[\frac{8+45.25}{2}\right] - \left[\frac{4+32}{2}\right] = 26.62 - 18 = 8.62$

Therefore, the area of the portion where both circle intersect is $\pi - 6.96$

20.

a) Solution:

The sum of the vector given = a + b = $\left((2+4)i, (-7+3)j, (4-2)k\right) = 6i - 4j + 2k$

$|C| = \sqrt{36+16+4} = \sqrt{56}$

Therefore, the required unit vector is

$$\hat{C} = \frac{1}{|C|}\vec{C} = \frac{6}{\sqrt{56}}i - \frac{4}{\sqrt{56}}j + \frac{2}{\sqrt{56}}k$$

The angle θ between two vectors a and b is given by

$$cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

To find the value of $\vec{a}.\vec{b}$

$\vec{a}.\vec{b} = (2i - 7j + 4k)(4i + 3j - 2k) = 8 - 21 - 8 = 21$

To find the value of $|\vec{a}| = \sqrt{4 + 49 + 16} = \sqrt{69}$ and $|\vec{b}| = \sqrt{16 + 9 + 4} = \sqrt{29}$

Hence the angle is

$$cos\theta = \frac{21}{\sqrt{69}\sqrt{29}} = \theta = 61.5°$$

Hence, the angle between the two vectors is 61.5°

b) Solution:

Here $n_1 = 2\acute{i} + 3\acute{j} + 9\acute{k}$ and $d_1 = 12$ and $n_2 = 5\acute{i} + 5\acute{j} + 5\acute{k}$ and $d_2 = 10$

Hence using the relation

$r(n_1 + \lambda n_2) = d_1 + \lambda d_2$

$r\left(2\acute{i} + 3\acute{j} + 9\acute{k} + \lambda(5\acute{i} + 5\acute{j} + 5\acute{k})\right) = 12 + \lambda 10$

Where λ is some constant multiplying $(x\acute{i} + y\acute{j} + x\acute{k})$ to $r\left(2\acute{i} + 3\acute{j} + 9\acute{k} + \lambda(5\acute{i} + 5\acute{j} + 5\acute{k})\right)$ we get

$(x\acute{i} + y\acute{j} + x\acute{k})\left(2\acute{i} + 3\acute{j} + 9\acute{k} + \lambda(5\acute{i} + 5\acute{j} + 5\acute{k})\right) = 12 + \lambda 10$

$(x\acute{i} + y\acute{j} + x\acute{k})\left((\lambda 5 + 2)\acute{i} + (\lambda 5 + 3)\acute{j} + (\lambda 5 + 9)\acute{k}\right) = 12 + \lambda 10$

$(\lambda 5 + 2)\acute{x} + (\lambda 5 + 3)y + (\lambda 5 + 9)z = 12 + \lambda 10$

Putting the value of x, y , z = 3, 3, 3. We get

$\lambda 5(x\acute{i} + y\acute{j} + x\acute{k}) - 10 + (2x + 3y + 9z - 12)$

$5(9\lambda - 2) + (30) = 0$

$45\lambda - 10 + 30 = 0$

$$45\lambda + 20 = 0; \lambda = -\frac{20}{45}$$

Putting the value of $\lambda = -\frac{20}{45}$ in $r\left((2\acute{i} + 3\acute{j} + 9\acute{k}) + -\frac{20}{45}(5\acute{i} + 5\acute{j} + 5\acute{k})\right) = 12 + \lambda 10$

$$r\left((2\acute{i} + 3\acute{j} + 9\acute{k}) - \left(\frac{20}{9}\acute{i} - \frac{20}{9}\acute{j} - \frac{20}{9}\acute{k}\right)\right) = 12 - \frac{20}{45}10$$

$r\left(-\frac{2}{9}i + \frac{7}{9}j + \frac{61}{9}k\right) = 7.55$ is the required vector.